Mathematics
Pre-Course Work for Double Maths course

cambridge area 14-19 partnership

Welcome to 6th Form Mathematics. This booklet is designed to prepare you for studying Mathematics post-16 by focusing on those topics from the GCSE/Level 2 Further Maths courses which have strong links to Key Stage 5 Mathematics.

This work is important. Please ensure that you complete if fully and bring solutions to your first Mathematics lesson. To ensure your readiness for the Double Maths course you will have a test on the content covered in this work at the beginning of term. The results may play a part in your final acceptance to the course.

Here is a list of resources you may find useful for guidance or additional practice:

HegartyMaths	https://www.hegartymaths.com
Mymaths	www.mymaths.co.uk
GCSE Bitesize	www.bbc.co.uk/schools.gcsebitesize/maths/algebra
MangaHigh	www.mangahigh.com/en_gb/maths_games/algebra
Maths Made Easy	www.mathsmadeeasy.co.uk/algebra/algebragcse.htm

Good luck!

You may use a scientific calculator whilst completing this work.

ALGEBRA:

1. Simplify

a) $25^{\frac{1}{2}}$ b) $27^{\frac{1}{3}}$ c) $\left(1\frac{9}{16}\right)^{\frac{3}{2}}$ d) $\left(\frac{3}{4}\right)^{0}$ e) $\frac{6^{-1}}{5}$ f) $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

g) $9^{-\frac{1}{2}}$ h $(-5)^{-3}$

2. Rationalise the denominator

a) $\frac{1}{2+\sqrt{5}}$ b) $\frac{4}{3-\sqrt{5}}$ c) $\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ d) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}-\sqrt{3}}$ e) $\frac{3-\sqrt{7}}{3+\sqrt{7}}$

3. Simplify the following algebraic fractions

a) $\frac{x^2+7x+12}{16-x^2}$ b) $\frac{3x^2-5x-12}{x^2-9}$ c) $\frac{2x-1}{x+1} \div \frac{2x^2-x+1}{x^2+3x+2}$ d) $\frac{3x^2-27}{x+2} \div \frac{x^2-6x+9}{x^2+x-2}$

e) $\frac{-4x^2+6x^4-2x}{-2x}$ f) $\frac{x^2+6x+8}{3x^2+7x+2}$ g) $\frac{2x^2-5x-3}{2x^2-9x+9}$

4. Solve the following equations by completing the square:

a) $x^2+4x-2=0$ b) $4x^2-x=8$ c) $5x^2+8x-2=0$

5. Solve the following equations:

a) $\frac{2}{x}-\frac{5}{2x-1}=0$ b) $\frac{3x+2}{2}-\frac{x-1}{5}=3$ c) $\frac{2}{3x-1}+\frac{1}{x+8}=\frac{1}{2}$

6. Sketch the following quadratics (indicating on every sketch):

- all points of intersection with the axes
- turning point (vertex; min/max)
- line of symmetry

a) $y=x^2-4x-12$ b) $y=x^2+11$ c) $y=14+5x-x^2$

d) $y=x^2+2x+7$ e) $y=x^2-5x+8$

7. By using Factor Theorem, show that $(x+4)$ is a factor of $f(x)=5x^3-73x+28$.

8. $(x-4)$ is a factor of $f(x)=x^3+2x^2+ax-76$. Find a.

9. $(x-1)$ and $(x+3)$ are both linear factors

   of $g(x) = x^3 + px^2 + qx + 6$.

   a) What is the third linear factor of $g(x)$?
   b) Find of p and q.

10. **(*)** By using the long division or otherwise factorise each of the expressions as a product of three linear factors.

a) $x^3 - 7x - 6$ b) $x^3 + 5x^2 + 3x - 9$ c) $x^3 + 7x^2 + 14x + 8$

11. Solve:

a). $\begin{matrix} 3x + 8y = 33 \\ 6x = 3 + 5y \end{matrix}$ b) $\begin{matrix} 4x - 5y = 4 \\ 6x + 2y = 25 \end{matrix}$ c) $\begin{matrix} x + y = 11 \\ xy = 30 \end{matrix}$

d) $\begin{matrix} 2x + 3y = 13 \\ x^2 + y^2 = 78 \end{matrix}$ e) $\begin{matrix} x - 2y = 1 \\ 3xy - y^2 = 8 \end{matrix}$

12. Solve the following inequalities

a) $12x - 3(x-3) < 45$ b) $x^2 - 2x - 3 < 0$

c). $x^2 - 5x - 14 > 0$ d) $(2x-3)(x+2) > 3(x-2)$

13. How can you tell whether a quadratic equation has any real roots and how many there are?

14. How can you check whether a quadratic expression is factorisable?

COORDINATE GEOMETRY *Lines and Circles*:

1. Write the equation of a line crossing the points A(1,3) and B(5,1). Write the answer in the form of $ax + by + c = 0$, where a, b and c are all integers.
2. Write the equation of a line perpendicular to the line $y + 0.5x + 3 = 0$ and crossing a point A(3,7).

3. The line $y = 2x - 10$ meets the x-axis at point A.

   The line $y = -2x + 4$ meets the y-axis at point B. Find the equation of the line joining A and B.

4. Find an equation of a line that passes through the point (-2,7) and is parallel to the line y=4x+1.
5. The line joining (c,4) and (7,6) has gradient $\frac{3}{4}$. Work out the value of c,
6. The line *l* passes through the points (-3,0) and (3,-2) and the line *n* passes through the points (1,8) and (-1,2). Show that the lines *l* and *n* are perpendicular.
7. Find an equation of a straight line passing through the points with coordinates (-1,5) and (4,-2).
   The line crosses the x-axis at point A and the y-axis at point B. Find the area of a triangle A0B (where 0 is the origin).
8. The line segment AB is a diameter of a circle, where A and B are (0,-2) and (6,-5) respectively. Show that the centre of the circle lies on the line $x - 2y - 10 = 0$.
9. The line AB is a diameter of a circle centre (4,-2). The line *l* passes through B and is perpendicular to AB. Given that A is (-2,6):
   a) find the coordinates of B
   b) find the equation of line *l*.
10. What is the centre and radius of each of the following circles (HINT: in b) and c) start by completing the square for *x* and for *y*):
    a) $(x - 5)^2 + (y + 4)^2 = 81$
    b) $x^2 + y^2 + 4x + 9y + 3 = 0$
    c) $2x^2 + 2y^2 + 8x + 15y - 1 = 0$
11. The point P(1,-2) lies on the circle centre (4,6).
    a) Find the equation of the circle
    b) Find the equation of a tangent to the circle at P.
12. Show that the line $x + y = 11$ is a tangent to the circle $x^2 + (y - 3)^2 = 32$
13. Show that the line $y = x - 10$ does not meet the circle $(x - 2)^2 + y^2 = 25$.
14. The points U(-2,8), V(7,7) and W(-3,-1) lie on a circle.
    a) Show that a triangle UVW has a right angle.
    b) Find the coordinates of the centre of the circle.

TRIGONOMETRY *Sine/Cosine Ruler; Area of a Triangle*

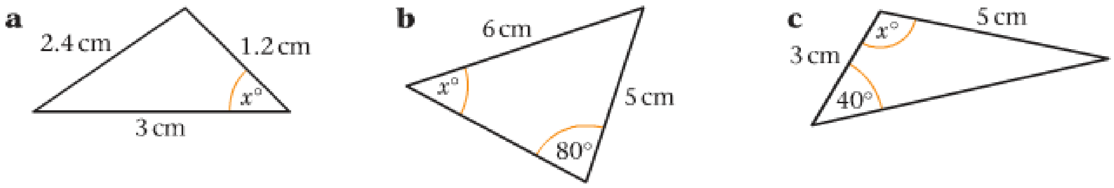
(Give non-exact answers to 3 significant figures.)

1 The area of a triangle is 10 cm². The angle between two of the sides, of length 6 cm and 8 cm respectively, is obtuse. Work out:

a The size of this angle.

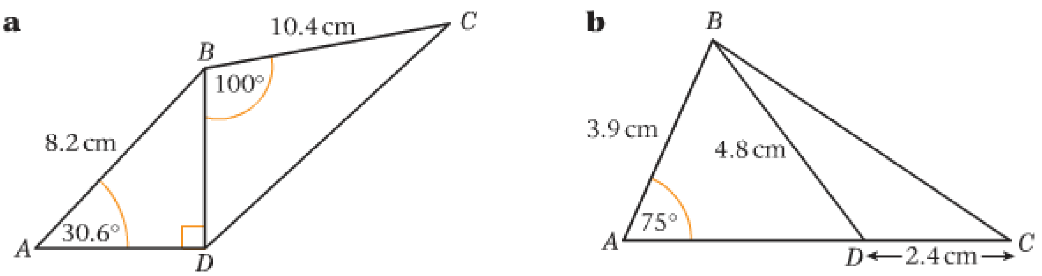
b The length of the third side.

2 In each triangle below, find the value of x and the area of the triangle:



3 The sides of a triangle are 3 cm, 5 cm and 7 cm respectively. Show that the largest angle is 120°, and find the area of the triangle.

4 In each of the figures below calculate the total area:



5 In $\triangle ABC$, $AB = 10$ cm, $BC = a\sqrt{3}$ cm, $AC = 5\sqrt{13}$ cm and $\angle ABC = 150°$. Calculate:

a The value of a.

b The exact area of $\triangle ABC$.

6 In a triangle, the largest side has length 2 cm and one of the other sides has length $\sqrt{2}$ cm. Given that the area of the triangle is 1 cm², show that the triangle is right-angled and isosceles.

7 The three points A, B and C, with coordinates $A(0, 1)$, $B(3, 4)$ and $C(1, 3)$ respectively, are joined to form a triangle:

a Show that $\cos \angle ACB = -\frac{4}{5}$.

b Calculate the area of $\triangle ABC$.

8 The longest side of a triangle has length $(2x - 1)$ cm. The other sides have lengths $(x - 1)$ cm and $(x + 1)$ cm. Given that the largest angle is 120°, work out:

a the value of x and **b** the area of the triangle.

FURTHER TRIGONOMETRY ***Identities, Equations, Graphs***

$sin^2x + cos^2x \equiv 1$	$tanx \equiv \frac{sinx}{cosx}$

1. Prove the following identities:
   a) $tanx\sqrt{1 - sin^2x} \equiv sinx$

   b) $\frac{1-cos^2x}{1-sin^2x} \equiv tan^2x$

   c) $\frac{2sinxcosx}{tanx} \equiv 2 - 2sin^2x$

2. Prove the following identities:

a $(\sin\theta + \cos\theta)^2 \equiv 1 + 2\sin\theta\cos\theta$

b $\frac{1}{\cos\theta} - \cos\theta \equiv \sin\theta\tan\theta$

c $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x\cos x}$

d $\cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$

e $(2\sin\theta - \cos\theta)^2 + (\sin\theta + 2\cos\theta)^2 \equiv 5$

f $2 - (\sin\theta - \cos\theta)^2 \equiv (\sin\theta + \cos\theta)^2$

g $\sin^2 x\cos^2 y - \cos^2 x\sin^2 y = \sin^2 x - \sin^2 y$

3. Solve the following equations in given intervals:
   a) $2sinx = -1\,,\ -360^o \leq x \leq 360^o$
   b) $\sqrt{2}cosx = 1,\ -270^o \leq x \leq 270^o$
   c) $sinxcosx = 0,\ 0^o \leq x \leq 540^o$
   d) $tanx = -\frac{1}{3},\ 0^o \leq x \leq 360^o$
   e) $sin^2x = \frac{3}{4},\ 0^o \leq x \leq 360^o$

4. **(*)** Solve the following equations for $0^o \leq x \leq 360^o$
   a) $2sinx + cosx = 0$
   b) $\sqrt{3}tanx = 2sinx$
   c) $sin^2x + cosx + 1 = 0$
   d) $2sin^2x = 3(1 - cosx)$
   e) $tanx = cosx$
   f) $4cosx(cosx - 1) = -5cosx$
   g) $2tan^2x + tanx - 1 = 0$

CALCULUS:

1. Differentiate the following with respect to x:

a) $f(x) = 3x - 7x^6$

b) $g(x) = \frac{x^4+7x^2}{x}$

c) $y = (3x+1)(2x-9)$

d) $y = x^{\frac{1}{2}}\left(x^{\frac{3}{2}} - x^3\right)$

e) $h(x) = \frac{\sqrt{x}-x^5}{2x^3}$

f) $y = \frac{2x^3+3x}{\sqrt{x}}$

2. Find the equation of a tangent to the curve at a given point:

a) $y = \frac{2x-1}{x}$, (1,1)

b) $y = x^2 - \frac{7}{x^2}$, (1,-6)

c) $y = 2x^3 + 6x + 10$, (-1,2)

3. Find the equation of a normal to the curve at a given point:

a) $y = x^2 - \frac{8}{\sqrt{x}}$, (4,12)

b) $y = x^2 - 5x$, (6,6)

4. For $(x) = 12 - 4x + 2x^2$, find an equation of a tangent and a normal at the point where $x = -1$.

5. Find a gradient of the curve $y = \left(x^{\frac{3}{2}} - 1\right)\left(x^{\frac{1}{2}} + 1\right)$ at the point where $x = 4$.

6. The curve with equation $y = ax^2 + bx + c$ passes through the point (1,2). The gradient of the curve at the point (2,1) is zero. Find the values of a, b and c.

7. The normals to the curve $2y = 3x^3 - 7x^2 + 4x$, at the points 0(0,0) and A(1,0), meet at point N.

   a) Find the coordinates of the point N.

   b) Find the area of a triangle 0AN.

8. Find the coordinates of the points on the curve with equation $y = x^3 - 11x + 1$ where the gradient is 1.

9. The total surface area of a cylinder $A\ cm^2$ with a fixed volume of 1000 cubic cm is given by the formula $A = 2\pi x^2 + \frac{200}{x}$, where x cm is the

radius of the base of the cylinder. Show that when the rate of change of the area with respect to the radius is zero, $x^3 = \frac{500}{\pi}$.

10. The volume, $V\ cm^3$, of a tin of radius $r\ cm$ is given by the formula $V = \pi(40r - r^2 - r^3)$. Find the positive value of r for which $\frac{dV}{dr} = 0$, and find the value o$f\ V$ which corresponds with this value or r.

NOTE: Questions with **(*)** provide an extension opportunity to the work you covered in class in Y11.

Please note that there is no requirement for you to complete those unless you are up for an extra challenge.